

Chapter Fourteen

Ratio, Similarity and Symmetry

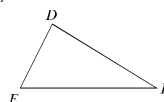
For comparing two quantities, their ratios are to be considered. Again, for determining ratios, the two quantities are to be measured in the same units. In algebra we have discussed this in detail.

At the end of this chapter, the students will be able to

- Explain geometric ratios
- Explain the internal division of a line segment
- Verify and prove theorems related to ratios
- Verify and prove theorems related to similarity
- Explain the concepts of symmetry
- Verify line and rotational symmetry of real objects practically

14.1 Properties of ratio and proportion

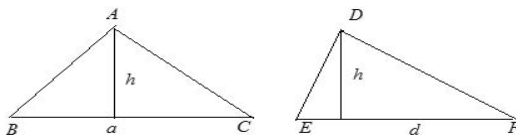
- (i) If $a : b = x : y$ and $c : d = x : y$, it follows that $a : b = c : d$.
- (ii) If $a : b = b : a$, it follows that $a = b$
- (iii) If $a : b = x : y$, it follows that $b \div a = y \div x$ (inversendo)
- (iv) If $a : b = x : y$, it follows that $a \div x = b \div y$ (alternendo)
- (v) If $a : b = c : d$, it follows that $ad = bc$ (cross multiplication)
- (vi) If $a : b = x : y$, it follows that $a + b \div b = x + y \div y$ (componendo)
and $a - b \div b = x - y \div y$ (dividendo)
- (vii) If $\frac{a}{b} = \frac{c}{d}$, it follows that $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo dividendo)



Geometrical Proportion

Earlier we have learnt to find the area of a triangular region. Two necessary concepts of ratio are to be formed from this.

(1) If the heights of two triangles are equal, their bases and areas are proportional.

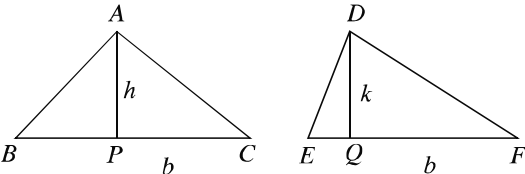


Let the bases of the triangles ABC and DEF be $BC = a$, $EF = d$ respectively and the height in both cases be h .

Hence, the area of the triangle $ABC = \frac{1}{2}a \times h$ and the area of the triangle $DEF = \frac{1}{2}d \times h$.

Therefore, area of the triangle ABC : area of the triangle $DEF = \frac{1}{2}a \times h : \frac{1}{2}d \times h$
 $= a : d = BC : EF$ that is, the areas and bases are proportional.

(2) **If the bases of two triangles are equal, their heights and areas are proportional.**



Let the heights of the triangles ABC and DEF be $AP = h$, $DQ = k$ respectively and the base in both cases be b . Hence, the area of the triangle $ABC = \frac{1}{2}b \times h$ and the area of the triangle $DEF = \frac{1}{2}b \times k$

Therefore, area of the triangle ABC : area of the triangle $DEF = \frac{1}{2}b \times h : \frac{1}{2}b \times k$
 $= h : k = AP : DQ$

Theorem 1
A straight line drawn parallel to one side of a triangle intersects the other two sides or those sides produced proportionally.

Proposition : In the figure, the straight line DE is parallel to the side BC of the triangle ABC . DE intersects AB and AC or their produced sections at D and E respectively. It is required to prove that, $AD : DB = AE : EC$.

Construction: Join B, E and C, D .

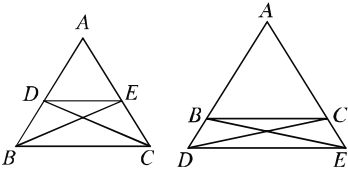


Fig1

Fig2

Proof:

Steps	Justification
(i) The heights of $\triangle ADE$ and $\triangle BDE$ are equal.	[The bases of the triangles of equal height are proportional]

$\therefore \frac{\Delta ADE}{\Delta BDE} = \frac{AD}{DB}$ <p>(2) Again, The heights of ΔADE and ΔDEC are equal.</p> $\therefore \frac{\Delta ADE}{\Delta DEC} = \frac{AE}{EC}$ <p>(3) $\Delta BDE = \Delta DEC$</p> $\therefore \frac{\Delta ADE}{\Delta BDE} = \frac{\Delta ADE}{\Delta DEC}$ <p>(4) Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$</p> <p>i.e., $AD \div DB = AE \div EC$.</p>	<p>[The bases of the triangles of equal height are proportional] [On the same base and between same pair of lines]</p>
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Corollary 1. If the line parallel to BC of the triangle ABC intersects the sides AB and AC at D and E respectively, $\frac{AB}{AD} = \frac{AC}{AE}$ and $\frac{AB}{BD} = \frac{AC}{CE}$.

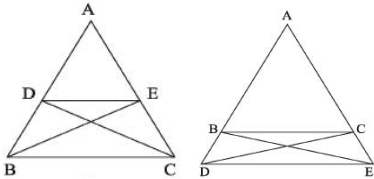
Corollary 2. The line through the mid point of a side of a triangle parallel to another side bisects the third line.

The proposition opposite of theorem 1 is also true. That is, if a line segment divides the two sides of a triangle or the line produced proportionally it is parallel to the third side. Here follows the proof of the theorem.

Theorem 2

If a line segment divides the two sides or their produced sections of a triangle proportionally, it is parallel to the third side.

Proposition : In the triangle ABC the line segment DE divides the two sides AB and AC or their produced sections proportionally. That is, $AD : DB = AE : EC$. It is required to prove that DE and BC are proportional.



Construction: Join B, E and C, D .

Proof:

Steps	Justification
(i) $\frac{\Delta ADE}{\Delta BDE} = \frac{AD}{DB}$	[Triangles with equal height] [Triangles with equal height]
and $\frac{\Delta ADE}{\Delta DEC} = \frac{AE}{EC}$	[given] [From (i) and (ii)]

$$(2) \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$(3) \text{ Therefore, } \frac{\triangle ADE}{\triangle BDE} = \frac{\triangle ADE}{\triangle DEC}$$

$$\therefore \triangle BDE = \triangle DEC$$

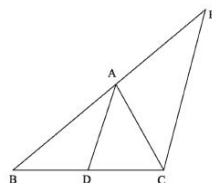
(4) $\triangle BDE$ and $\triangle DEC$ are on the same side of the common base DE . \therefore they lie between a pair of parallel lines. Hence BC and DE are parallel.

Theorem 3

The internal bisector of an angle of a triangle divides its opposite side in the ratio of the sides constituting to the angle.

Proposition : In $\triangle ABC$ the line segment AD bisects the internal angle $\angle A$ and intersects the side BC at D . It is required to prove that $BD : DC = BA : AC$.

Construction: Draw the line segment CE parallel to DA , so that it intersects the side BA produced at E .



Proof:

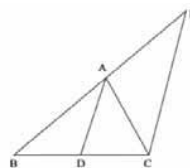
Steps	Justification
(1) Since $DA \parallel CE$ and both BC and AC are their transversal and $\angle AEC = \angle BAD$ and $\angle ACE = \angle CAD$	[by construction] [corresponding angles] [alternate angles] [supposition]
(2) $\therefore \angle BAD = \angle CAD$ $\therefore \angle AEC = \angle ACE$; $\therefore AC = AE$	[Theorem 1] [step (2)]
(3) Again, since $DA \parallel CE$, $\therefore \frac{BD}{DC} = \frac{BA}{AE}$	
(4) $\therefore AE = AC$ $\therefore \frac{BD}{DC} = \frac{BA}{AC}$	

Theorem 4

If any side of a triangle is divided internally, the line segment from the point of division to the opposite vertex bisects the angle at the vertex.

Proposition : Let ABC be a triangle and the line segment AD from vertex A divides the side BC at D such that $BD : DC = BA : AC$. It is required to prove that AD bisects $\angle BAC$, i.e. $\angle BAD = \angle CAD$.

Construction : Draw at C the line segment CE parallel to DA , so that it intersects the side BA produced at E .



Proof:

Steps	Justification
(1) In $\triangle BCE$, $DA \parallel CE$	[by construction]
$\therefore BA : AE = BD : DC$	[Theorem]
(2) But $BD : DC = BA : AC$	[supposition]
$\therefore BA : AE = BA : AC$	[from steps (1) and (2)]
$\therefore AE = AC$	[Use angles of isosceles are equal]
Therefore, $\angle ACE = \angle AEC$	[Corresponding angles]
(4) But $\angle AEC = \angle BAD$	[alternate angles]
and $\angle ACE = \angle CAD$	[from step (2)]
Therefore, $\angle BAD = \angle CAD$	
i.e., the line segment AD bisects $\angle BAC$.	

Exercise 14.1

- The bisectors of two base angles of a triangle intersect the opposite sides at X and Y respectively. If XY is parallel to the base, prove that the triangle is an isosceles triangle.
- Prove that if two lines intersect a few parallel lines, the matching sides are proportional.
- Prove that the diagonals of a trapezium are divided in the same ratio at their point of intersection.
- Prove that the line segment joining the mid points of oblique sides of a trapezium and two parallel sides are parallel.
- The medians AD and BE of the triangle ABC intersect each other at G . A line segment is drawn through G parallel to DE which intersects AC at F . Prove that $AC = 3EF$.
- In the triangle ABC , X is any point on BC and O is a point on AX . Prove that $\triangle AOB : \triangle AOC = BX : XC$
- In the triangle ABC , the bisector of $\angle A$ intersects BC at D . A line segment drawn parallel to BC intersects AB and AC at E and F respectively. Prove that $BD : DC = BE : CF$.

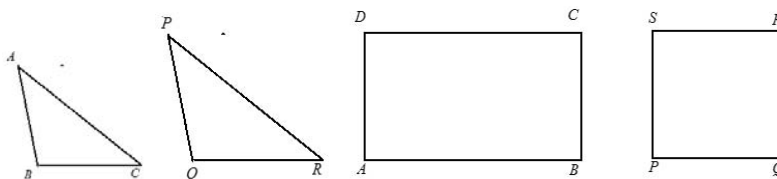
8. If the heights of the equiangular triangles ABC and DEF are AM and DN respectively, prove that $AM : DN = AB : DE$.

14.2 Similarity

The congruence and similarity of triangles have been discussed earlier in class VII. In general, congruence is a special case of similarity. If two figures are congruent, they are similar; but two similar triangles are not always congruent.

Equiangular Polygons:

If the angles of two polygons with equal number of sides are sequentially equal, the polygons are known as equiangular polygons.



Equiangular triangle

Equiangular quadrilateral

Similar Polygons:

If the vertices of two polygons with equal number of sides can be matched in such a sequential way that

- (i) The matching angles are equal
- (ii) The ratios of matching sides are equal, the two polygons are called similar polygons.

In the above figures, the rectangle $ABCD$ and the square $PQRS$ are equiangular since the number of sides in both the figures is 4 and the angles of the rectangle are sequentially equal to the angles of the square (all right angles). Though the similar angles of the figure are equal, the ratios of the matching sides are not the same. Hence the figures are not similar. In case of triangles, situation like this does not arise. As a result of matching the vertices of triangles, if one of the conditions of similarity is true, the other condition automatically becomes true and the triangles are similar. That is, similar triangles are always equiangular and equiangular triangles are always similar.

If two triangles are equiangular and one of their matching pairs is equal, the triangles are congruent. The ratio of the matching sides of two equiangular triangles is a constant. Proofs of the related theorems are given below.

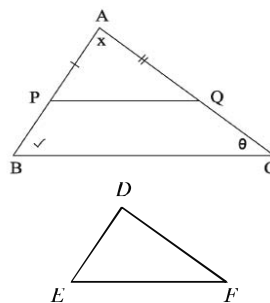
Theorem 5

If two triangles are equiangular, their matching sides are proportional.

Proposition : Let ABC and DEF be triangles with $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. We need to prove that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Construction: Consider the matching sides of the triangles ABC and DEF unequal. Take two points P and Q on AB and AC respectively so that $AP = DE$ and $AQ = DF$. Join P and Q and complete the construction.

Proof:



Steps	Justification
<p>(1) In the triangles APQ and DEF $AP = DE$, $AQ = DF$, $\angle A = \angle D$ Therefore, $\triangle APQ \cong \triangle DEF$ Hence, $\angle APQ = \angle DEF = \angle ABC$ and $\angle AQP = \angle DFE = \angle ACB$. That is, the corresponding angles produced as a result of intersections of AB and AC by the line segment PQ are equal.</p> <p>Therefore, $PQ \parallel BC$; $\therefore \frac{AB}{AP} = \frac{AC}{AQ}$ or,</p> $\frac{AB}{DE} = \frac{AC}{DF}.$	<p>[SAS theorem]</p> <p>[theorem 1]</p>
<p>(2) Similarly, cutting line segments ED and EF from B and C respectively, it can be shown that i.e., $\frac{BA}{ED} = \frac{BC}{EF}$ i.e., $\frac{AB}{DE} = \frac{BC}{EF}$; $\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.</p>	<p>[theorem 1]</p>

The proposition opposite of theorem 5 is also true.

Theorem 6

If the sides of two triangles are proportional, the opposite angles of their matching sides are equal.

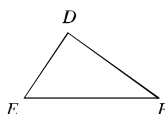
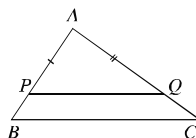
Proposition : If in $\triangle ABC$ and $\triangle DEF$,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}.$$

It is to prove that,

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F.$$

Construction: Consider the matching sides of the triangles ABC and DEF unequal. Take two points P and Q on AB and AC respectively so that $AP = DE$ and $AQ = DF$. Join P and Q .



Proof:

Steps	Justification
<p>(1) Since $\frac{AB}{DE} = \frac{AC}{DF}$, so, $\frac{AB}{AP} = \frac{AC}{AQ}$.</p> <p>Therefore, $PQ \parallel BC$</p> <p>$\therefore \angle ABC = \angle APQ$ and $\angle ACB = \angle AQP$</p> <p>\therefore Triangles ABC and APQ are equiangular.</p> <p>Therefore, $\frac{AB}{AP} = \frac{BC}{PQ}$, so, $\frac{AB}{DE} = \frac{BC}{AQ}$.</p> <p>$\therefore \frac{BC}{EF} = \frac{BC}{PQ}$ [supposition]; $\therefore \frac{AB}{DE} = \frac{BC}{EF}$</p> <p>$\therefore EF = PQ$</p> <p>Therefore, $\triangle APQ$ and $\triangle DEF$ are congruent.</p> <p>\therefore</p> <p>$\angle PAQ = \angle EDF, \angle APQ = \angle DEF, \angle AQP = \angle DFE,$</p> <p>$\therefore \angle APQ = \angle ABC$ and $\angle AQP = \angle ACB$</p> <p>$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F.$</p>	<p>[Theorem 2]</p> <p>Corresponding angles made by the transversal AB</p> <p>Corresponding angles made by the transversal AC</p> <p>[Theorem 3]</p> <p>[SSS Theorem]</p>

Theorem 7

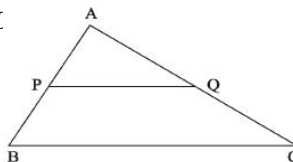
If one angle of a triangle is equal to an angle of the other and the sides adjacent to the equal angles are proportional, the triangles are similar.

Proposition : If in $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D$

and $\frac{AB}{DE} = \frac{AC}{DF}$.

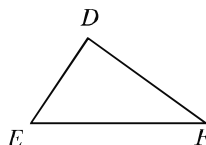
It is to be proved that the triangles $\triangle ABC$ and $\triangle DEF$ are similar.

Construction: Consider the matching sides of $\triangle ABC$ and $\triangle DEF$ unequal. Take two points P and



Q on AB and AC respectively so that $AP = DE$ and $AQ = DF$. Join P and Q .

Proof:



Steps	Justification
(1) In $\triangle APQ$ and $\triangle DEF$, $AP = DE$, $AQ = DF$ and included $\angle A =$ included $\angle D$ $\therefore \triangle APQ \cong \triangle DEF$ $\therefore \angle A = \angle D$, $\angle APQ = \angle E$, $\angle AQP = \angle F$. (2) Again, $\frac{AB}{DE} = \frac{AC}{DF}$, so $\frac{AB}{AP} = \frac{AC}{AQ}$ $\therefore PQ \parallel BC$ Therefore, $\angle ABC = \angle APQ$ and $\angle ACB = \angle AQP$ $\therefore \angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ i.e., triangles ABC and DEF are equiangular. Therefore $\triangle ABC$ and $\triangle DEF$ are similar.	[SAS Theorem] [Theorem 2]

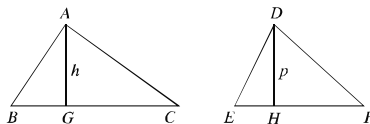
Theorem 8

The ratio of the areas of two similar triangles is equal to the ratio of squares on any two matching sides.

Proposition : Let the triangles ABC and DEF be similar and BC and EF be their matching sides respectively. It is required to prove that $\triangle ABC \sim \triangle DEF \Rightarrow BC^2 \sim EF^2$

Construction: Draw perpendiculars AG and DH on BC and EF respectively. Let $AG = h$, $DH = p$.

Proof:



Steps	Justification
(1) $\triangle ABC = \frac{1}{2} BC \cdot h$ and $\triangle DEF = \frac{1}{2} EF \cdot p$ $\therefore \frac{\triangle ABC}{\triangle DEF} = \frac{\frac{1}{2} BC \cdot h}{\frac{1}{2} EF \cdot p} = \frac{h \cdot BC}{p \cdot EF} = \frac{h}{p} \times \frac{BC}{EF}$ (2) In the triangles ABG and DEH , $\angle B = \angle E$, $\angle AGB = \angle DHE$ (\angle right angle)	

$$\therefore \angle BAG = \angle EDH$$

(3) because $\triangle ABC$ and $\triangle DEF$ are similar.

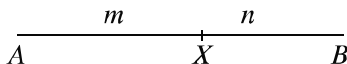
$$\frac{h}{p} = \frac{AB}{DE} = \frac{BC}{EF}$$

$$\therefore \frac{\triangle ABC}{\triangle DEF} = \frac{h}{p} \times \frac{BC}{EF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$$

[Triangles $\triangle ABC$ and $\triangle DEF$ are similar]

14.1 Internal Division of a Line Segment in definite ratio

If A and B are two different points in a plane and m and n are two natural numbers, we acknowledge that there exists a unique point X lying between A and B and $AX : XB = m : n$.



In the above figure, the line segment AB is divided at X internally in the ratio $m : n$, i.e. $AX : XB = m : n$.

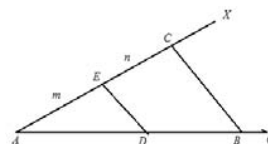
Construction 1

To divide a given line segment internally in a given ratio.

Let the line segment AB be divided internally in the ratio $m : n$.

Construction: Let an angle $\angle BAX$ be drawn at A . From AX cut the lengths $AE = m$ and $EC = n$ sequentially.

Join B, C . At E , draw line segment ED parallel to CB which intersects AB at D . Then the line segment AB is divided at D internally in the ratio $m : n$.



Proof: Since the line segment DE is parallel to a side BC of the triangle ABC

$$\therefore AD : DB = AE : EC = m : n.$$

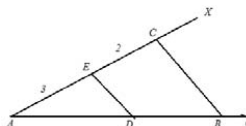
Activity :

1. Divide a given line segment in definite ratio internally by an alternative method.

Example 1. Divide a line segment of length 7cm internally in the ratio 3:2.

Solution: Draw any ray AG . From AG , cut a line segment $AB = 7\text{cm}$. Draw an angle $\angle BAX$ at A .

From AX , cut the lengths $AE = 3\text{ cm}$ and $EC = 2\text{ cm}$. from EX . Join B, C . At E , draw an angle $\angle AED$ equal to $\angle ACB$ whose side intersects AB at D . Then the line segment AB is divided at D internally in the ratio 3:2.

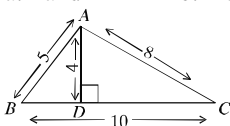


Exercise 14.2

1. Consider the following information:
- ratios are considered to compare two expressions
 - to find ratio, expressions are measured in the same unit
 - to find ratio, expressions must be of the same type.

Which case of the following is true?

- a. i and ii b. ii and iii c. i and iii d. i, ii and iii



Use the information from the above figure to answer the questions 2 and 3.

2. What is the ratio of the height and base of the triangle ABC ?

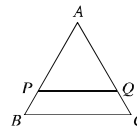
- a. $\frac{1}{2}$ b. $\frac{4}{5}$ c. $\frac{2}{5}$ d. $\frac{5}{4}$

3. What is the area of triangle ABD in sq. units?

- a. 6 b. 20 c. 40 d. 6

4. In triangle ABC , if $PQ \parallel BC$, which of the following is true?

- a. $AP : PB = AQ : QC$ b. $AB : PQ = AC : PQ$
c. $AB : AC = PQ : BC$ d. $PQ : BC = BP : BQ$



5. In a square how many lines of symmetry are there?

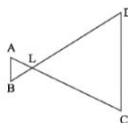
- a. 10 b. 8 c. 6 d. 4

6. Prove that if each of the two triangles is similar to a third triangle, they are congruent to each other.

7. Prove that, if one acute angle of a right angled triangle is equal to an acute angle of another right angled triangle, the triangles are similar.

8. Prove that the two right angled triangles formed by the perpendicular from the vertex containing the right angle are similar to each other and also to the original triangle.

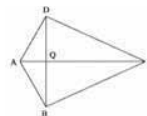
9. In the adjacent figure, $\angle B = \angle D$ and $CD = 4 AB$. Prove that $BD = 5 BL$.



10. A line segment drawn through the vertex A of the parallelogram $ABCD$ intersects the BC and DC at M and N respectively. Prove that $BM \times DN$ is a constant.

11. In the adjacent figure, $BD \perp AC$ and

$DQ = BA = 2AQ = \frac{1}{2}QC$. $BD = 5BL$. Prove that, $DA \perp DC$.



12. In the triangles ABC and DEF $\angle A = \angle D$. Prove that,
 $\Delta ABC \sim \Delta DEF \Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$.
13. The bisector AD of $\angle A$ of the triangle ABC intersects BC at D . The line segment CE parallel to DA intersects the line segment BA extended.
- Draw the specified figure.
 - Prove that $BD : DC = BA : AC$.
 - If a line segment parallel to BC intersect AB and AC at P and Q respectively, prove that $BD : DC = BP : CQ$.

14. In the figure, ABC and DEF are two similar triangles.

- Write the matching sides and matching angles of the triangles.

- Prove that,

$$\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$



- If $BC = 3$ cm, $EF = 8$ cm, $\angle B = 60^\circ$, $\frac{BC}{AB} = \frac{3}{2}$ and $\angle ABC = 3$ sq cm,

draw the triangle DEF and find its area.

14.4 Symmetry

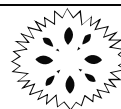
Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of our activity. Artists, designers, architects, carpenters always make use of the idea of symmetry. The treeleaves, the flowers, the beehives, houses, tables, chairs -everywhere we find symmetrical designs. A figure has line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.



Each of the above figures has the line of symmetry. The last figure has two lines of symmetry

Activity:

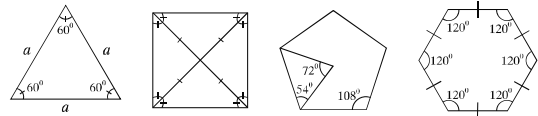
- Sumi has made some papereut design as shown in the adjacent figure. In the figure, mark the lines of symmetry. How many lines of symmetry does the figure have?
- Write and identify the letters in English alphabet having line symmetry. Also mark their line of symmetry.



14.5 Lines of Symmetry of Regular Polygons

A polygon is a closed figure made of several line segments. A polygon is said to be regular if all its sides are of equal length and all its angles are equal. The triangle is a

polygon made up of the least number of line segments. An equilateral triangle is a regular polygon of three sides. An equilateral triangle is regular because its sides as well as angles are equal. A square is the regular polygon of four sides. The sides of a square are equal and each of the angles is equal to one right angle. Similarly, in regular pentagons and hexagons, the sides are equal and the angles are equal as well.



Each regular polygons is a figure of symmetry. Therefore, it is necessary to know their lines of symmetry. Each regular polygon has many lines of symmetry as it has many sides.

Three lines of symmetry	Four lines of symmetry	Five lines of symmetry	Six lines of symmetry
Equilateral triangle	Square	Regular pentagon	Regular hexagon

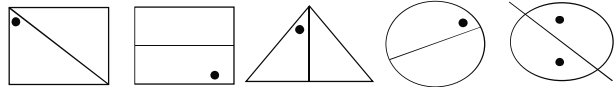
The concept of line symmetry is closely related to mirror reflection. A geometrical figure, has line symmetry when one half of it is the mirror image of the other half. So, the line of symmetry is also called the reflection symmetry.



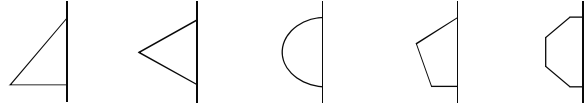
Exercise 14.3

1. Which of the following figures have line symmetry?
(a) A house (b) A mosque (c) A temple (d) A church (e) A pagoda
(f) Parliament house (g) The Tajmahal.

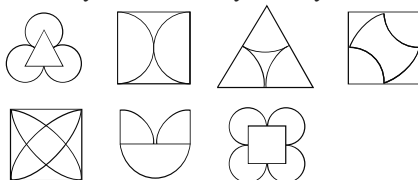
2. The line of symmetry is given, find the other hole:



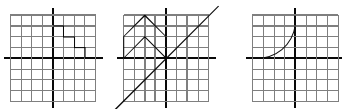
3. In the following figures, the line of symmetry is given; complete and identify the figure.



4. Identify the lines of symmetry in the following geometrical figures:



5. Complete each of the following incomplete geometrical shapes to be symmetric about the mirror line:



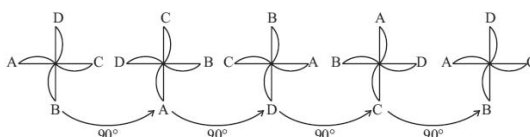
6. Find the number of lines of symmetry of the following geometrical figures:
- An isosceles triangle
 - A scalene triangle
 - A square
 - A rhombus
 - A pentagon
 - A regular hexagon
 - A circle
7. Draw the letters of the English alphabet which have reflection symmetry with respect to
- a vertical mirror
 - a horizontal mirror
 - both horizontal and vertical mirrors.
8. Draw three examples of shapes with no line of symmetry.

14.6 Rotational Symmetry

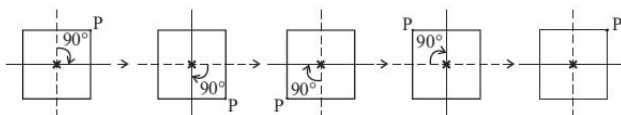
When an object rotates around any fixed point, its shape and size do not change. But the different parts of the object change their position. If the new position of the object after rotation becomes identical to the original position, we say the object has a **rotational symmetry**. The wheels of a bicycle, ceiling fan, square are examples of objects having rotational symmetry etc.. As a result of rotation the blades of the fan looks exactly the same as the original position more than once. The blades of a fan may rotate in the clockwise direction or in the anticlockwise direction. The wheels of a bicycle may rotate in the clockwise direction or in the anticlockwise direction. The rotation in the anticlockwise direction is considered the positive direction of rotation.

This fixed point around which the object rotates is the **centre of rotation**. The angle of turning during rotation is called the **angle of rotation**. A full-turn means rotation by 360° ; a half-turn is rotation by 180° .

In the figure below, a fan with four blades rotating by 90° is shown in different positions. It is noted that us a full turn of the four positions (rotating about the angle by 90° , 180° , 270° and 360°), the fan looks exactly the same. For this reason, it is said that the rotational symmetry of the fan is order 4.



Here is one more example for rotational symmetry. Consider the intersection of two diagonals of a square the centre of rotation. In the quarter turn about the centre of the square, any diagonal position will be as like as the second figure. In this way, when you complete four quarterturns, the square reaches its original position. It is said that a square has a **rotational symmetry of order 4**.

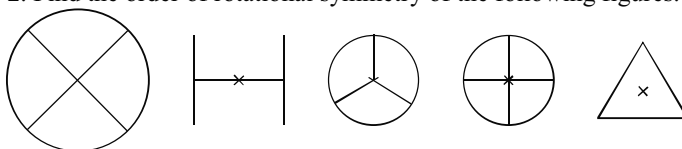


Observe also that every object occupies same position after one complete revolution. Every geometrical object has a rotational symmetry of order 1. Such cases have no interest for us. For finding the rotational symmetry of an object, one need to consider the following matter.

- The centre of rotation
- The angle of rotation
- The direction of rotation
- The order of rotational symmetry.

Activity:

- Give examples of 5 plane objects from your surroundings which have rotational symmetry.
- Find the order of rotational symmetry of the following figures.



14-7 Line Symmetry and Rotational Symmetry

We have seen that some geometrical shapes have only line symmetry, some have only rotational symmetry and some have both line symmetry and rotational symmetry. For example, the square has four lines of symmetry as well rotational symmetry of order 4

The circle is the most symmetrical figure, because it can be rotated around its centre through any angle. Therefore, it has unlimited order of rotational of symmetry. At the same time, every line through the centre forms a line of reflection symmetry and so it has unlimited number of lines of symmetry.

Activity:

1. Determine the line of symmetry and the rotational symmetry of the given alphabet and complete the table below:

Letter	Line of symmetry	Number of lines of symmetry	Rotational symmetry	Order of rotational symmetry
Z	↯	0	↻	2
H				
O				
E				
C				

Exercise 14-4

1. Find the rotational symmetry of the following figures:

(a)

(b)

(c)

(d)

(e)

(f)

2. When you slice a lemon the crosssection looks as shown in the figure. Determine the rotational symmetry of the figure.

3. Fill in the blanks:

Shape	Centre of Rotation	Order of Rotation	Angle of Rotation
Square			
Rectangle			
Rhombus			
Equilateral triangle			
Circle			
Regular pentagon			

4. Name the quadrilaterals which have line of symmetry and rotational symmetry of order more than 1.

5. When we have a rotational symmetry of a body of order more than 1 whose angle of rotation is 180°. Justify your answer.